

Extended Goldstone–boson–exchange constituent–quark model

K. Glantschnig, R. Kainhofer, W. Plessas^a, B. Sengl, and R.F. Wagenbrunn

Theoretical Physics, Institute of Physics, University of Graz, Universitätsplatz 5, A-8010 Graz, Austria

Received: 6 September 2004 / Revised version: 25 October 2004 /
Published online: 3 January 2005 – © Società Italiana di Fisica / Springer-Verlag 2005
Communicated by Th. Walcher

Abstract. We present an extension of the Goldstone–boson–exchange constituent–quark model including additional interactions beyond the ones used hitherto. For the hyperfine interaction between the constituent quarks we assume pseudoscalar, vector, and scalar meson exchanges and consider all relevant force components produced by these types of exchanges. The resulting model, which corresponds to a relativistic Poincaré-invariant Hamiltonian (or equivalently mass operator), provides a unified framework for a covariant description of all light and strange baryons. The ground states and resonances up to an excitation energy of about 2 GeV are reproduced in fair agreement with phenomenology, with the exception of the first excitations above the Λ and Ξ ground states.

PACS. 12.39.Ki Relativistic quark model – 14.20.-c Baryons (including antiparticles)

1 Introduction

It is nowadays widely accepted that quantum chromodynamics (QCD) is the basic theory of strong interactions. Over the years, however, one has learned that QCD manifests itself with different degrees of freedom in different energy regimes. As a consequence one resorts to separate approaches for the solution of QCD depending on the energy domain one is working in. Low-energy QCD is characterized by the appearance of constituent quarks as quasiparticles (see, *e.g.*, the lattice studies in ref. [1]). Therefore, it seems to be appropriate to treat hadrons in terms of constituent quarks. An effective tool towards a comprehensive description of low-energy hadron phenomena is offered by constituent–quark models (CQMs). They can be designed to incorporate the essential ingredients of low-energy QCD. At the same time they can make use of a covariant formulation so as to allow for the necessary relativistic treatment of hadrons as composite systems of constituent quarks confined to a relatively small volume. When building a CQM one faces the problem of the proper effective interaction between the constituent quarks. While the confinement can readily be modeled after lattice results of QCD (see, *e.g.*, ref. [2]), there are alternative approaches to the hyperfine interaction of the constituent quarks.

One type of hyperfine interaction between constituent quarks derives from one-gluon exchange (OGE) [3]. Several CQMs are based on this dynamical concept [4–8]. While OGE CQMs have a long tradition, all of them are

facing some serious problems especially in baryon spectroscopy. In particular, they do not manage to describe in a uniform manner the level schemes in the N , Δ , and Λ spectra in accordance with experimental data [9].

Another approach to the hyperfine interaction of constituent quarks considers instanton-induced effects [10–12]. The CQM based on this concept also encounters similar difficulties in the N and Δ spectra as it cannot reproduce the correct level orderings of positive- and negative-parity excitations [13].

Congruent with the assumption of constituent quarks as the essential degrees of freedom at low energies is the appearance of Goldstone bosons. Therefore, it seems to be quite natural to derive the hyperfine interaction from Goldstone–boson exchange (GBE) [14]. A CQM based on this dynamical concept has indeed been quite successful in describing the excitation spectra of the light and strange baryons [15]. The specific spin-flavor symmetry that is brought about by GBE allows to reproduce in a unified framework specifically the characteristic patterns in the level schemes of the N , Δ , and Λ spectra [9, 15].

The CQM of ref. [15] uses only a part of the dynamics offered by GBE, namely, only the spin–spin part of the pseudoscalar exchange. Obviously, one is interested in the performance of a more complete model that takes into account also the other force components of the single Goldstone–boson exchange, such as the complete tensor force, as well as the possibilities offered by multiple GBE [16]. First preliminary attempts in this direction were already made in refs. [17–20]. In the present paper we report an extended version of the GBE CQM that includes

^a e-mail: plessas@uni-graz.at

all relevant force components offered by pseudoscalar, vector, and scalar exchanges.

We also address the role of spin-orbit forces in more detail. Spin-orbit forces can stem from both the confinement and the hyperfine interactions. From phenomenology, however, one expects their net effect to be small in baryon spectroscopy. The more this should be true for quadratic spin-orbit forces. Therefore, in CQMs it is often assumed that spin-orbit as well as quadratic spin-orbit forces can be left out. Nevertheless, we study the influences of spin-orbit forces by considering two versions of the extended GBE CQM, one without and one with such type of interaction.

This work is organized as follows: In sect. 2 we shortly recapitulate the properties of the pseudoscalar GBE CQM of ref. [15], which is restricted to spin-spin forces of pseudoscalar exchange only. In sect. 3 we discuss the idea of multiple GBE and its effective description. The formulation of the extended GBE CQM is presented in sect. 4. In sect. 5 we give the parameterizations of the two versions of the extended GBE CQM constructed here. The resulting spectra of all the light and strange baryons are presented in sect. 6. We close with a discussion of the obtained results and an outlook to possible applications and tests of the new type of GBE CQM in low-energy hadronic physics.

2 Pseudoscalar Goldstone-boson-exchange constituent-quark model

In ref. [15] one constructed a CQM relying on the Hamiltonian

$$H = \sum_{i=1}^3 \sqrt{\mathbf{p}_i^2 + m_i^2} + \sum_{i<j} [V_{\text{conf}}(i, j) + V_{\text{hf}}(i, j)], \quad (1)$$

where \mathbf{p}_i are the three-momenta of the constituent quarks and m_i are their masses. The confinement potential V_{conf} was chosen to be of a linear form, and the hyperfine potential V_{hf} consisted of the spin-spin components of the pseudoscalar meson exchange (π, K, η, η'). The Hamiltonian (1) corresponds to an invariant (interacting) mass operator M in relativistic quantum mechanics and it lends itself to covariant calculations not only of the energy spectra but also of further hadronic reactions.

The spectra and wave functions of the light and strange baryons were obtained by solving the eigenvalue problem of the above Hamiltonian (or equivalently of the mass operator M) via the stochastic variational method (SVM) [21, 22]. This approach provides a very reliable determination of the eigenenergies and eigenstates. The accuracy of the method was checked before by solving the problem of a relativistic three-quark system via (modified) Faddeev equations [23].

The spectra for the light and strange baryons resulting from the pseudoscalar GBE CQM are shown in fig. 1. As is immediately evident, quite a satisfactory description of the spectroscopy of all light and strange baryons can be

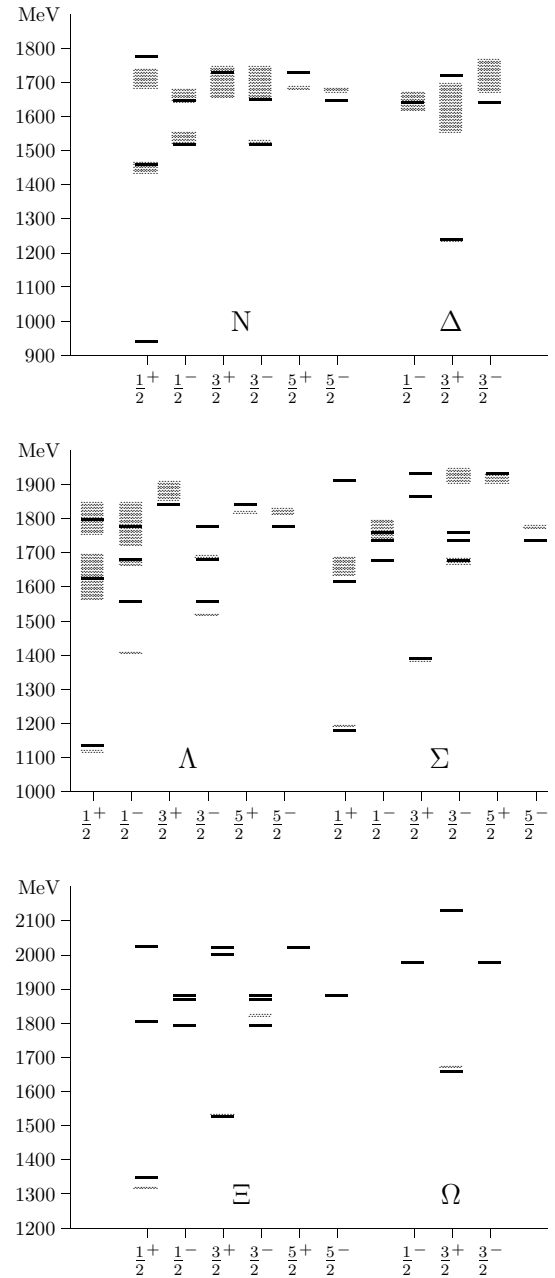


Fig. 1. Energy levels of the lowest light- and strange-baryon states for the GBE CQM of ref. [15]. The mass of the nucleon ground state is 939 MeV. The shadowed boxes represent the experimental values together with their uncertainties [24].

achieved already with this kind of GBE CQM. Especially, the orderings of the energy levels according to their parities are obtained in agreement with experiment. In the spectrum of the nucleon, the Roper resonance $N(1440)$ with $J^P = \frac{1}{2}^+$ comes out as the lowest excitation above the nucleon ground state. It is followed by the $\frac{1}{2}^-$ - $\frac{3}{2}^-$ doublet $N(1535)$ - $N(1520)$. In the spectra of the Λ and the Σ the $\frac{1}{2}^+$ resonances $\Lambda(1600)$ and $\Sigma(1660)$ fall below the resonances with negative parity $\Lambda(1670)$, $\Lambda(1690)$,

and $\Sigma(1750)$. In addition, the two $\frac{1}{2}^- - \frac{3}{2}^-$ states $\Lambda(1405)$ - $\Lambda(1520)$ remain the lowest excitations above the ground state. This behavior rests on the explicit flavor dependence of the GBE CQM and is governed by the particular spin-flavor symmetry coming with GBE.

With the specific type of hyperfine interaction deriving from GBE the intricate problem of the correct ordering of the low-lying excitations of the light and strange baryons is thus readily resolved. However, there remain some other problems. Most strikingly the $\Lambda(1405)$ -resonance remains far from the experimental value. Furthermore, some states in the bands of higher excitations in the N (as well as the Λ) spectrum do not fit the experimental data; especially the near degeneracies of the $\frac{1}{2}^+ - \frac{1}{2}^-$ and similarly of the $\frac{3}{2}^+ - \frac{3}{2}^-$ and $\frac{5}{2}^+ - \frac{5}{2}^-$ N -resonances at about 1700 MeV are not reproduced.

In this situation an obvious question was if further improvements of the description of the light- and strange-baryon spectra could be obtained by an extension of the pseudoscalar GBE CQM to including further force components. Wagenbrunn *et al.* made first attempts to extend the GBE CQM and proceeded to study the influences of the tensor forces from pseudoscalar GBE [17,18]. However, with these tensor forces alone no improvement could be achieved, because the remarkably small splittings in the various multiplets of alike total-angular-momentum states could not be reproduced as observed in experiment. From phenomenology one expects the total tensor-force effects to be small. Therefore one went ahead to include also vector- and scalar-type exchanges in addition to the pseudoscalar one [17]. The spin-orbit forces, however, had then not yet been taken into account in the parameterization of an extended GBE CQM [19,20].

3 Multiple Goldstone-boson exchange

In the pseudoscalar GBE CQM a reasonable description of the light- and strange-baryon spectra is achieved only if the hyperfine interaction is based solely on the spin-spin part. If the tensor forces are included (with a strength as prescribed by unique octet and singlet pseudoscalar coupling constants, with values $g_{ps,8}^2/4\pi = 0.67$ and $g_{ps,0}^2/4\pi = 0.9$, respectively, see ref. [15]), one can no longer keep certain splittings small, as demanded by phenomenology. This is true especially for the $\frac{1}{2}^- - \frac{3}{2}^-$ resonances at around 1510 MeV as well as the $\frac{1}{2}^- - \frac{3}{2}^- - \frac{5}{2}^-$ resonances at around 1650 MeV. The behavior is demonstrated in fig. 2, where in panel b the effects of the tensor forces due to pseudoscalar meson exchange are demonstrated; obviously the generated splittings are too large especially in the second band.

In the picture of GBE dynamics there is also the possibility of multiple-boson exchanges, in addition to the single-boson exchange prevailing in the ansatz of pseudoscalar coupling [16]. One may take the effects of such multiple-boson exchanges into account by foreseeing single-boson exchanges of scalar and vector type, just as

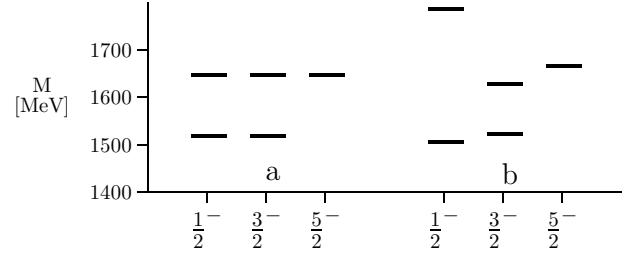


Fig. 2. LS multiplets of nucleon excitation levels for the pseudoscalar GBE CQM: a) without tensor forces (as in fig. 1), b) with tensor forces.

in the meson-exchange model of the nucleon-nucleon interaction [25,26]. For example, the effects of two-pion exchanges can effectively be described by σ -meson exchange ($\pi\pi$ S -wave) and by ρ -meson exchange ($\pi\pi$ P -wave), the effects of three-pion exchanges by ω -meson exchange, and so forth. One may assume that an analogous approach works in the case of Goldstone-boson-exchange dynamics between constituent quarks.

4 Extended Goldstone-boson-exchange constituent-quark model

Motivated by the discussion above we constructed our extended version of the GBE CQM. For this purpose we considered pseudoscalar, vector, and scalar meson exchanges, as well as all relevant force components (*i.e.* central, spin-spin, tensor, and spin-orbit forces). The Hamiltonian of the extended GBE CQM has the following form:

$$H = \sum_{i=1}^3 \sqrt{\mathbf{p}_i^2 + m_i^2} + \sum_{i<j} [V_{\text{conf}}(i,j) + V_{\text{hf}}(i,j)]. \quad (2)$$

Again \mathbf{p}_i are the three-momenta and m_i are the masses of the constituent quarks. $V_{\text{conf}}(i,j)$ is the confinement potential and $V_{\text{hf}}(i,j)$ denotes the hyperfine interaction between quarks i and j . It is now taken to be the sum of the pseudoscalar (ps), vector (v), and scalar (s) meson-exchange potentials,

$$\begin{aligned} V_{\text{hf}}(i,j) &= V^{\text{ps}}(i,j) + V^{\text{v}}(i,j) + V^{\text{s}}(i,j) \\ &= \sum_{a=1}^3 [V_{\pi}(i,j) + V_{\rho}(i,j) + V_{a_0}(i,j)] \lambda_i^a \cdot \lambda_j^a \\ &\quad + \sum_{a=4}^7 [V_K(i,j) + V_{K^*}(i,j) + V_{\kappa}(i,j)] \lambda_i^a \cdot \lambda_j^a \\ &\quad + [V_{\eta}(i,j) + V_{\omega_8}(i,j) + V_{f_0}(i,j)] \lambda_i^8 \cdot \lambda_j^8 \\ &\quad + \frac{2}{3} [V_{\eta'}(i,j) + V_{\omega_0}(i,j) + V_{\sigma}(i,j)], \end{aligned} \quad (3)$$

where λ_i^a denote the Gell-Mann flavor matrices.

Subsequently we shall give the explicit expressions for the phenomenological confinement potential as well as for

the spatial parts of the different meson-exchange potentials using the following form factor for the constituent quark-meson vertex [20]:

$$F(\mathbf{q}^2) = \frac{\Lambda_\gamma^2 - \mu_\gamma^2}{\Lambda_\gamma^2 + \mathbf{q}^2}. \quad (4)$$

Here, \mathbf{q} is the three-momentum of the exchanged meson γ , μ_γ its mass, and Λ_γ the corresponding cut-off parameter. For mesons with different masses these cut-off parameters may be different. Their values should be bigger for mesons with larger masses.

4.1 Confinement

For the confinement interaction we assume a linear potential dependent on the distance r_{ij} between constituent quarks i and j

$$V_{\text{conf}}(i, j) = V_0 + Cr_{ij}. \quad (5)$$

The strength C is treated as a fit parameter but it turns out to be of a magnitude roughly corresponding to the string tension of QCD. The constant V_0 is needed to fix the ground-state energy of the spectrum, *i.e.* the nucleon, to its phenomenological value of 939 MeV.

According to lattice QCD calculations [2] one would expect an additional (short-range) Coulomb term of the form $-a/r_{ij}$ (with strength a) in the confinement potential. Our studies, however, have revealed that such a term can effectively be absorbed into the remaining potential parts. In particular, its effect can be compensated by a slight adjustment of the linear term Cr_{ij} and a small variation of the cut-off parameter Λ_σ in the scalar exchange potential [27]. Thus, in all our calculations we just employ the linear confinement potential (5).

4.2 Pseudoscalar part

The pseudoscalar meson-exchange interaction ($\gamma = \pi, K, \eta, \eta'$) produces spin-spin and tensor forces. The corresponding potential is

$$V_\gamma(i, j) = V_\gamma^{\text{SS}}(\mathbf{r}_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + V_\gamma^{\text{T}}(\mathbf{r}_{ij}) [3(\hat{r}_{ij} \cdot \boldsymbol{\sigma}_i)(\hat{r}_{ij} \cdot \boldsymbol{\sigma}_j) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j], \quad (6)$$

with V_γ^{SS} the spin-spin and V_γ^{T} the tensor components; $\boldsymbol{\sigma}_i$ denotes the spin operator of quark i . The dependences of V_γ^{SS} and V_γ^{T} on the interquark separation $\mathbf{r}_{ij} = \mathbf{r}$ are given as follows:

Spin-spin component:

$$V_\gamma^{\text{SS}}(\mathbf{r}) = \frac{g_\gamma^2}{4\pi} \frac{1}{12m_i m_j} \left[\mu_\gamma^2 \frac{e^{-\mu_\gamma r}}{r} - \left(\mu_\gamma^2 + \frac{\Lambda_\gamma(\Lambda_\gamma^2 - \mu_\gamma^2)r}{2} \right) \frac{e^{-\Lambda_\gamma r}}{r} \right]; \quad (7)$$

Tensor component:

$$V_\gamma^{\text{T}}(\mathbf{r}) = \frac{g_\gamma^2}{4\pi} \frac{1}{12m_i m_j} \left[\mu_\gamma^2 \left(1 + \frac{3}{\mu_\gamma r} + \frac{3}{\mu_\gamma^2 r^2} \right) \frac{e^{-\mu_\gamma r}}{r} - \Lambda_\gamma^2 \left(1 + \frac{3}{\Lambda_\gamma r} + \frac{3}{\Lambda_\gamma^2 r^2} \right) \frac{e^{-\Lambda_\gamma r}}{r} - \frac{(\Lambda_\gamma^2 - \mu_\gamma^2)(1 + \Lambda_\gamma r)e^{-\Lambda_\gamma r}}{2r} \right]. \quad (8)$$

Here, g_γ represents the quark-meson coupling constant. Again, m_i are the constituent-quark masses, μ_γ the meson masses, and Λ_γ the corresponding cut-offs.

4.3 Vector part

The vector meson-exchange interaction ($\gamma = \rho, K^*, \omega_8, \omega_0$) produces central, spin-spin, tensor, as well as spin-orbit forces. The corresponding potential is

$$V_\gamma(i, j) = V_\gamma^{\text{C}}(\mathbf{r}_{ij}) + V_\gamma^{\text{SS}}(\mathbf{r}_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + V_\gamma^{\text{T}}(\mathbf{r}_{ij}) [3(\hat{r}_{ij} \cdot \boldsymbol{\sigma}_i)(\hat{r}_{ij} \cdot \boldsymbol{\sigma}_j) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j] + V_\gamma^{\text{LS}}(\mathbf{r}_{ij}) \mathbf{L}_{ij} \cdot \mathbf{S}_{ij}, \quad (9)$$

where \mathbf{L}_{ij} and \mathbf{S}_{ij} are the two-body angular-momentum and spin operators. The dependences of V^{C} , V^{SS} , V^{T} , and V^{LS} on the interquark separation $\mathbf{r}_{ij} = \mathbf{r}$ are given as follows:

Central component:

$$V_\gamma^{\text{C}}(\mathbf{r}) = \frac{(g_\gamma^{\text{V}})^2}{4\pi} \left[\frac{e^{-\mu_\gamma r}}{r} - \left(1 + \frac{(\Lambda_\gamma^2 - \mu_\gamma^2)r}{2\Lambda_\gamma} \right) \frac{e^{-\Lambda_\gamma r}}{r} \right]; \quad (10)$$

Spin-spin component:

$$V_\gamma^{\text{SS}}(\mathbf{r}) = 2 \frac{(g_\gamma^{\text{V}} + g_\gamma^{\text{T}})^2}{4\pi} \frac{1}{12m_i m_j} \left[\mu_\gamma^2 \frac{e^{-\mu_\gamma r}}{r} - \left(\mu_\gamma^2 + \frac{\Lambda_\gamma(\Lambda_\gamma^2 - \mu_\gamma^2)r}{2} \right) \frac{e^{-\Lambda_\gamma r}}{r} \right]; \quad (11)$$

Tensor component:

$$V_\gamma^{\text{T}}(\mathbf{r}) = -\frac{(g_\gamma^{\text{V}} + g_\gamma^{\text{T}})^2}{4\pi} \frac{1}{12m_i m_j} \times \left[\mu_\gamma^2 \left(1 + \frac{3}{\mu_\gamma r} + \frac{3}{\mu_\gamma^2 r^2} \right) \frac{e^{-\mu_\gamma r}}{r} - \Lambda_\gamma^2 \left(1 + \frac{3}{\Lambda_\gamma r} + \frac{3}{\Lambda_\gamma^2 r^2} \right) \frac{e^{-\Lambda_\gamma r}}{r} - \frac{(\Lambda_\gamma^2 - \mu_\gamma^2)(1 + \Lambda_\gamma r)e^{-\Lambda_\gamma r}}{2r} \right]; \quad (12)$$

Spin-orbit component:

$$\begin{aligned}
V_\gamma^{\text{LS}}(\mathbf{r}) = & -\frac{(g_\gamma^{\text{V}})^2}{4\pi} \left(3 + 4\frac{g_\gamma^{\text{T}}}{g_\gamma^{\text{V}}} \right) \frac{1}{2m_i m_j} \\
& \times \left[\mu_\gamma^3 \left(\frac{1}{\mu_\gamma^2 r^2} + \frac{1}{\mu_\gamma^3 r^3} \right) e^{-\mu_\gamma r} \right. \\
& \left. - \Lambda_\gamma^3 \left(\frac{1}{\Lambda_\gamma^2 r^2} + \frac{1}{\Lambda_\gamma^3 r^3} \right) e^{-\Lambda_\gamma r} - \frac{\Lambda_\gamma^2 - \mu_\gamma^2}{2r} e^{-\Lambda_\gamma r} \right].
\end{aligned} \tag{13}$$

The notation is the same as before, only we encounter different vector and tensor coupling constants g_γ^{V} and g_γ^{T} , respectively.

In principle, the vector meson exchange (like the subsequent scalar meson exchange) also produces a quadratic spin-orbit interaction. Since it is of higher order in the inverse quark masses, it is expected to be of minor importance. Therefore it is neglected here (and below in the scalar meson exchange).

4.4 Scalar part

The scalar meson-exchange interaction ($\gamma = a_0, \kappa, f_0, \sigma$) produces only central and spin-orbit forces. The corresponding potential is

$$V_\gamma(i, j) = V_\gamma^{\text{C}}(\mathbf{r}_{ij}) + V_\gamma^{\text{LS}}(\mathbf{r}_{ij}) \mathbf{L}_{ij} \cdot \mathbf{S}_{ij}. \tag{14}$$

The dependences of V^{C} and V^{LS} on the interquark separation $\mathbf{r}_{ij} = \mathbf{r}$ are given as follows:

Central component:

$$V_\gamma^{\text{C}}(\mathbf{r}) = -\frac{g_\gamma^2}{4\pi} \left[\frac{e^{-\mu_\gamma r}}{r} - \left(1 + \frac{(\Lambda_\gamma^2 - \mu_\gamma^2) r}{2\Lambda_\gamma} \right) \frac{e^{-\Lambda_\gamma r}}{r} \right]; \tag{15}$$

Spin-orbit component:

$$\begin{aligned}
V_\gamma^{\text{LS}}(\mathbf{r}) = & -\frac{g_\gamma^2}{4\pi} \frac{1}{2m_i m_j} \left[\mu_\gamma^3 \left(\frac{1}{\mu_\gamma^2 r^2} + \frac{1}{\mu_\gamma^3 r^3} \right) e^{-\mu_\gamma r} \right. \\
& \left. - \Lambda_\gamma^3 \left(\frac{1}{\Lambda_\gamma^2 r^2} + \frac{1}{\Lambda_\gamma^3 r^3} \right) e^{-\Lambda_\gamma r} - \frac{\Lambda_\gamma^2 - \mu_\gamma^2}{2r} e^{-\Lambda_\gamma r} \right].
\end{aligned} \tag{16}$$

The quadratic spin-orbit component is neglected (see the discussion in the previous subsection).

5 Parameterization

In this section we present two parameterizations of our extended GBE CQM. One parameterization leaves out spin-orbit forces. The other one takes into account all relevant force components produced by the different meson exchanges, and thus includes also spin-orbit forces.

Table 1. Predetermined parameters of the extended GBE CQM (for both cases, without and with spin-orbit forces). For additional explanations see the text.

$m_u = 340 \text{ MeV}$	$m_d = 340 \text{ MeV}$	$m_s = 507 \text{ MeV}$
$\mu_\pi = 139 \text{ MeV}$	$\mu_K = 494 \text{ MeV}$	$\mu_\eta = 547 \text{ MeV}$
$\mu_{\eta'} = 958 \text{ MeV}$	$\mu_\rho = 770 \text{ MeV}$	$\mu_{K^*} = 892 \text{ MeV}$
$\mu_{\omega_8} = 947 \text{ MeV}$	$\mu_{\omega_0} = 869 \text{ MeV}$	$\mu_\sigma = 680 \text{ MeV}$
$\mu_{a_0} = 980 \text{ MeV}$	$\mu_\kappa = 980 \text{ MeV}$	$\mu_{f_0} = 980 \text{ MeV}$
$g_{\text{ps},8}^2/4\pi = 0.67$	$(g_{\text{v},8}^{\text{V}})^2/4\pi = 0.55$	$(g_{\text{v},0}^{\text{V}})^2/4\pi = 1.107$
$(g_{\text{ps},0}^{\text{T}}/g_{\text{ps},8}^{\text{T}})^2 = 1$	$(g_{\text{v},8}^{\text{T}})^2/4\pi = 0.16$	$(g_{\text{v},0}^{\text{T}})^2/4\pi = 0.0058$
$g_s^2/4\pi = 0.67$		

5.1 Extended GBE CQM without spin-orbit forces

Usually it is expected that spin-orbit forces play only a minor role in hadron spectroscopy. Therefore they are left out in most CQMs. In extending the GBE CQM we may, in a first step, try this option too and set $V_\gamma^{\text{LS}}(\mathbf{r}_{ij}) = 0$ in eqs. (9) and (14).

In the parameterization of the various potential parts the constituent-quark masses m_i are set to the usual values adopted in CQMs. The meson masses are taken from the compilation of the Particle Data Group [24]. The mixing of the η_0 and η_8 mesons, whose mixing angle is -11.5° (as determined from the squares of the meson masses), was neglected in the previous pseudoscalar GBE CQM because the mixing effect had been found to be unimportant. We maintained this attitude in the parameterization of the extended models. The mixing of the vector ω_0 and ω_8 mesons, however, is much larger, namely 38.7° . Therefore, we took care of this mixing and assumed it to be ideal, *i.e.* with a mixing angle of 35.3° . No mixing was foreseen for the scalar mesons. We remark, however, that all of these assumptions for the meson masses are not very relevant. They have to be seen in the light of the values of the corresponding cut-off parameters Λ determined in the fit.

For the quark-meson coupling constants one may derive suitable estimates from the phenomenologically known π - N , ρ - N , and ω - N coupling constants using the Goldberger-Treiman relation. This procedure should lead to reasonable magnitudes at least for the coupling constants $g_{\text{ps}}^2/4\pi$ of pseudoscalar meson exchange as well as the vector and tensor coupling constants $(g_{\text{v}}^{\text{V}})^2/4\pi$ and $(g_{\text{v}}^{\text{T}})^2/4\pi$, respectively, of vector meson exchange (for details see ref. [28]). For the pseudoscalar coupling constants we have made the additional assumption that there is no difference between the octet and singlet exchanges, *i.e.*, $g_{\text{ps},8}^2/4\pi = g_{\text{ps},0}^2/4\pi$. For the scalar meson exchange we have assumed that the quark-meson coupling constant $g_s^2/4\pi$ is of equal magnitude as in the pseudoscalar case.

The numerical values of all the predetermined parameters are summarized in table 1.

In addition to the predetermined parameters, the extended GBE CQM (without spin-orbit forces) relies on seven fit parameters. Two of them concern the confinement interaction: the strength C of the linear potential

Table 2. Free parameters of the extended GBE CQM without spin-orbit forces.

$C = 1.935 \text{ fm}^{-2}$	$V_0 = -336 \text{ MeV}$	
$A_\pi = 834 \text{ MeV}$	$A_\rho = 1145 \text{ MeV}$	$A_\sigma = 1513 \text{ MeV}$
$A_K = 1420 \text{ MeV}$	$A_{\eta'} = 1400 \text{ MeV}$	

and the constant V_0 fixing the ground state of the spectrum. As indicated above, the value of C is found rather close to the magnitude of the string tension of QCD (of approximately 0.1 GeV^2). We remark that such a (strong) value for the strength of the linear confinement potential is compatible in a CQM only if the relativistic expression for the kinetic-energy operator is used. Otherwise, in a purely nonrelativistic CQM, this strength would have to be chosen unrealistically small [29].

The rest of the free parameters is furnished by the cut offs inherent in the meson-exchange potentials. They stem from the finite extension of the quark-meson vertices according to eq. (4). The various A_γ 's are expected to be different for the different meson exchanges. Instead of varying them freely, we prescribed a linear dependence of the cut-off parameters on the specific meson masses and adjusted only the parameters A_π , A_ρ , and A_σ occurring therein by a fit to the baryon spectra. Specifically, the different scaling prescriptions read:

Pseudoscalar meson exchange:

$$A_\gamma = A_\pi + (\mu_\gamma - \mu_\pi), \quad \gamma = \pi, \eta. \quad (17)$$

Vector meson exchange:

$$A_\gamma = A_\rho + (\mu_\gamma - \mu_\rho), \quad \gamma = \rho, K^*, \omega_8, \omega_0. \quad (18)$$

Scalar meson exchange:

$$A_\gamma = A_\sigma + (\mu_\gamma - \mu_\sigma), \quad \gamma = f_0, a_0, \kappa, \sigma. \quad (19)$$

Only A_K and $A_{\eta'}$ are exempted from this prescription and are varied independently. This turned out to be favorable for an optimal description of all the light- and strange-baryon excitation spectra.

The values of the seven free parameters of the extended GBE CQM (without spin-orbit forces) are summarized in table 2.

5.2 Extended GBE CQM with spin-orbit forces

In principle, spin-orbit forces are generated by both the confinement and the hyperfine interactions. Their net effect should be small, however, as one does not observe large level splittings from experiments. Still, we have aimed at a version of the extended GBE CQM that does include spin-orbit forces too. They might be relevant in applications beyond spectroscopy.

Instead of employing the explicit expressions for the $\mathbf{L} \cdot \mathbf{S}$ forces generated by the different meson exchanges in

Table 3. Free parameters of the extended GBE CQM with spin-orbit forces.

$C = 1.935 \text{ fm}^{-2}$	$V_0 = -336 \text{ MeV}$	
$A_\pi = 834 \text{ MeV}$	$A_\rho = 1145 \text{ MeV}$	$A_\sigma = 1513 \text{ MeV}$
$A_K = 1420 \text{ MeV}$	$A_{\eta'} = 1400 \text{ MeV}$	
$(g^{\text{LS}})^2/4\pi = 0.8$		

eqs. (13) and (16), we used a single spin-orbit term

$$V_\gamma^{\text{LS}}(\mathbf{r}) = -\frac{(g^{\text{LS}})^2}{4\pi} \frac{1}{2m_i m_j} \left[\mu_\gamma^3 \left(\frac{1}{\mu_\gamma^2 r^2} + \frac{1}{\mu_\gamma^3 r^3} \right) e^{-\mu_\gamma r} - \Lambda_\gamma^3 \left(\frac{1}{\Lambda_\gamma^2 r^2} + \frac{1}{\Lambda_\gamma^3 r^3} \right) e^{-\Lambda_\gamma r} - \frac{\Lambda_\gamma^2 - \mu_\gamma^2}{2r} e^{-\Lambda_\gamma r} \right],$$

$$(\gamma = \rho, K^*, \omega_8, \omega_0, f_0, a_0, \kappa, \sigma), \quad (20)$$

with a uniform strength $(g^{\text{LS}})^2/4\pi$, which is treated as an open parameter.

The assumption of a spin-orbit force as in eq. (20) is motivated by the following findings. The spin-orbit components in the hyperfine interaction are a-priori determined from the (vector and scalar) meson exchanges. In particular, their strengths are fixed by the corresponding quark-meson coupling constants. The spin-orbit forces from the confinement, however, remain uncertain in any case, both with respect to their form and strength. In this regard, one cannot escape to assume an *ad hoc* spin-orbit contribution in the parameterization of the quark-quark interaction. In fact, this problem has been studied in the work [28] by keeping the spin-orbit interactions of the meson exchanges from eqs. (13) and (16) fixed and varying the strength of the additional spin-orbit force from the confinement of the form as in eq. (20). One arrives at a certain combination of spin-orbit forces at the cost of additional open parameters. It was found that there is essentially no difference of such a version of an extended GBE CQM from the one presented here. Therefore, in our study of the role of spin-orbit forces we contented ourselves with the form as specified in eq. (20).

All the other interactions are kept the same as in the case of the extended GBE CQM of the previous subsection. Also the values of the predetermined parameters are maintained as given in table 1.

The extended GBE CQM with spin-orbit forces now relies on eight open parameters, two for the confinement and six for the hyperfine interaction. While the confinement parameters C and V_0 as well as the meson cut-offs Λ assume the same values as before (in the case without spin-orbit forces), one has an additional open parameter, namely, the strength $(g^{\text{LS}})^2/4\pi$ of the overall spin-orbit force (20). The values of the free parameters of the extended GBE CQM with spin-orbit forces are summarized in table 3.

6 Results

We are now presenting the spectra of the light and strange baryons produced by the two versions of the extended GBE CQM defined in the previous sections. For the N and Δ spectra all resonance levels are shown up to an excitation energy of about 1800 MeV. For the strange baryons all analogous octet and decuplet states are given, as well as the two lowest singlet states in the Λ spectrum. The comparison with experiment is made along the data compiled by the Particle Data Group [24]; only three- and four-star resonances with known J^P are considered.

6.1 Extended GBE CQM without spin-orbit forces

Figure 3 shows the spectra of the light and strange baryons as predicted by the extended GBE CQM without spin-orbit forces (see sect. 5.1). Evidently, the theoretical results (solid horizontal bars) are in overall good agreement with the experimental values (shown as gray boxes). In particular, the ground states are described reasonably well. Upon examining the N spectrum in more detail, one observes that the correct level ordering of positive- and negative-parity excitations is achieved in a satisfactory manner; the Roper resonance $N(1440)$ lies well below the first negative-parity resonance $N(1535)$. In addition, the level splittings within the experimentally almost degenerate multiplets remain generally small. In this respect the extended GBE CQM resembles the spectral properties of the pseudoscalar GBE CQM with spin-spin hyperfine interactions only (cf. fig. 1). However, for the present version of the extended GBE CQM (without spin-orbit forces), one also observes deficiencies with regard to the $\frac{5}{2}^+ N(1680)$ and $\frac{5}{2}^- N(1675)$ states; their practical degeneracy observed in experiment is not brought about. We require explicit spin-orbit forces in order to improve these levels (see the next subsection).

The strange baryons are reproduced reasonably well at least with respect to the ground states and some 4-star resonances. Like other CQMs, also the GBE CQM has difficulties in reproducing the Λ spectrum. While the level ordering of positive- and negative-parity states is in principle correct (namely, opposite to the N spectrum), the $\Lambda(1405)$ is missed by far. Obviously, this state cannot be described as a pure $\{QQQ\}$ state. It is strongly influenced by the nearby K - N decay threshold. A similar effect might still influence the $\Lambda(1520)$; it is predicted too high by about 60 MeV.

In the PDG listings one finds a $\Xi(1690)$ -resonance with a 3-star status, however, with uncertain J^P . It is therefore not shown in our figures. We find no theoretical level that could match such a state in the relevant energy region. The next excitations beyond the decuplet ground state $\Xi(1530)$ lie close to 1800 MeV. Here, one could also think about an influence of the K - Σ threshold. For instance, for the lowest $\frac{1}{2}^- \Xi$ a similar effect as for the $\frac{1}{2}^- \Lambda(1405)$ could occur and it could relate to the Ξ -resonance experimentally seen at an energy around 1690 MeV.

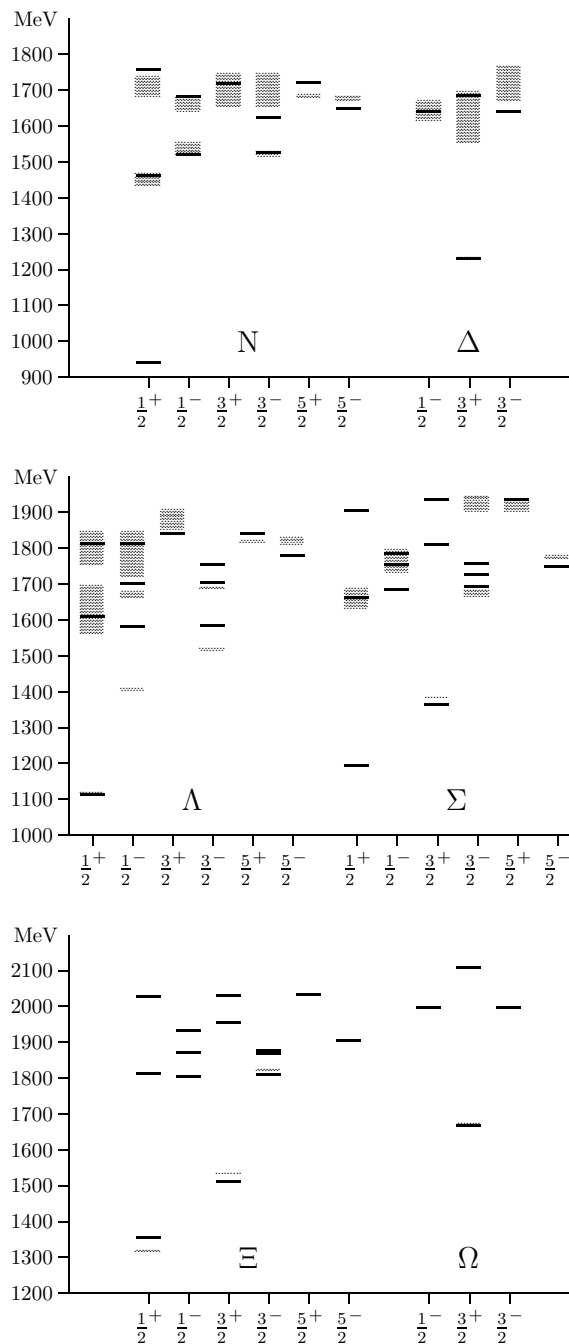


Fig. 3. Energy levels of the lowest light- and strange-baryon states for the extended GBE CQM without spin-orbit forces. The shadowed boxes represent the experimental values and their uncertainties [24].

6.2 Extended GBE CQM with spin-orbit forces

The spectra of the extended GBE CQM with spin-orbit forces (see sect. 5.2) are shown in fig. 4. In general, the spectral properties of this version are similar to the previous one but the almost degenerate levels $\frac{5}{2}^+ N(1680)$ and $\frac{5}{2}^- N(1675)$ in the N spectrum as well as the $\frac{5}{2}^+ \Lambda(1820)$

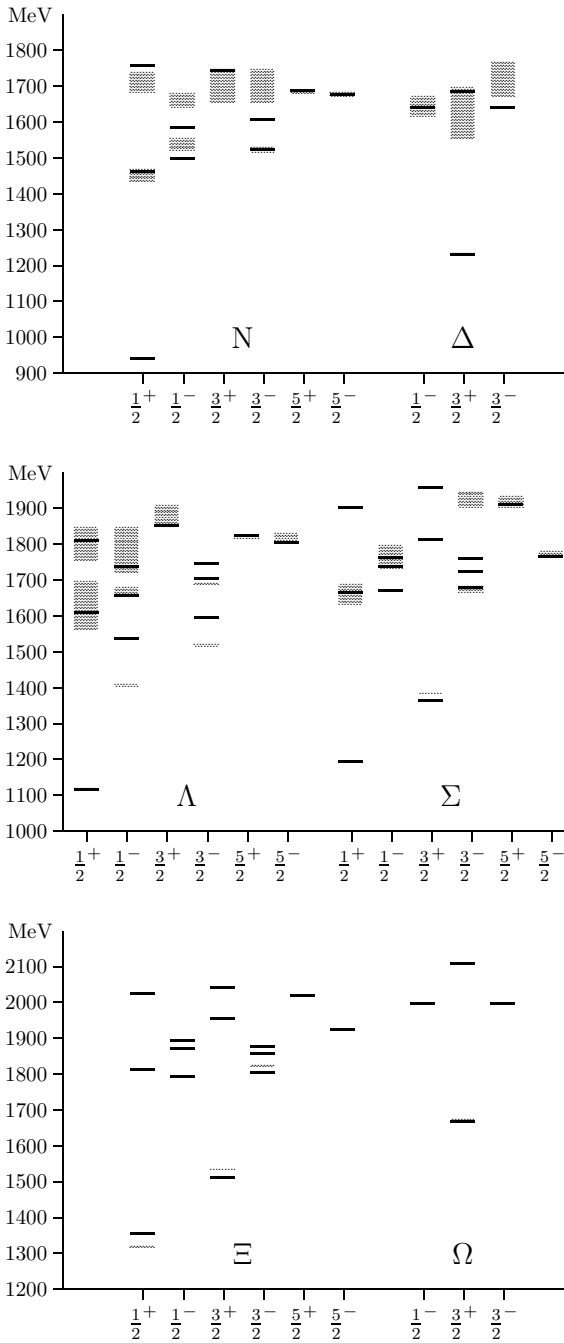


Fig. 4. Energy levels of the lowest light- and strange-baryon states for the extended GBE CQM including spin-orbit forces. The shadowed boxes represent the experimental values and their uncertainties [24].

and $\frac{5}{2}^- \Lambda(1830)$ in the Λ spectrum are now improved. The same is true for the $\frac{5}{2}^+ \Sigma(1915)$ and $\frac{5}{2}^- \Sigma(1775)$. The splitting of the $\frac{5}{2}^+ \Xi$ and $\frac{5}{2}^- \Xi$ gets reduced. These achievements are due to the additional spin-orbit forces. Such a behavior is not obtained in neither one of the other versions of the GBE CQM (cf. figs. 1 and 3).

7 Conclusions

The GBE CQM by the Graz group [15] so far relied on the spin-spin part of the pseudoscalar meson exchange only. Here, we have studied further possibilities offered by GBE dynamics for the hyperfine interaction of constituent quarks in light and strange baryons. In particular, we have taken into account the effects of multiple Goldstone-boson exchange through the exchanges of vector and scalar mesons in addition to the pseudoscalar ones. All relevant force components have been considered.

We have presented two versions of the extended GBE CQM, one without and one with spin-orbit forces. The relativistic quark model Hamiltonian involves only a handful of open parameters. The hyperfine interaction needs five fit parameters in the case without spin-orbit forces and six in the case with spin-orbit forces; the confinement interaction is always the same, namely, a linear potential whose strength is congruent with the one deduced from lattice QCD calculations. In both cases a reasonable description of the excitation spectra of all light and strange baryons can be achieved. The two versions differ in the quality of reproducing higher-lying resonance levels. The almost degeneracy of the $\frac{5}{2}^+ N(1680)$ and $\frac{5}{2}^- N(1675)$ levels in the N spectrum as well as the $\frac{5}{2}^+ \Lambda(1820)$ and $\frac{5}{2}^- \Lambda(1830)$ levels in the Λ spectrum can only be obtained in the case when spin-orbit forces are included. At the same time the description of the $\frac{5}{2}^+ \Sigma(1915)$ and $\frac{5}{2}^- \Sigma(1775)$ splitting is also improved.

The GBE dynamics is now comprehensively included in the quark model Hamiltonian for baryons. This Hamiltonian is equivalent to a covariant mass operator to be used in further studies within relativistic (*i.e.* Poincaré-invariant) quantum mechanics. Its eigenstates are readily obtained in the rest frame of any baryon. They can be boosted to an arbitrary reference frame in either one of the possible forms of relativistic quantum mechanics (instant, front, point forms etc.). Most economically (and accurately) this can be done in point form, where the Lorentz transformations remain interaction-free.

It will be most interesting to repeat the investigations of baryon reactions so far done with the pseudoscalar version of the GBE CQM, in particular, the calculations of the nucleon electroweak structure [30–33] and of the mesonic decays of the baryon resonances [34, 35]. One may expect new insights into the properties of the corresponding observables once all force components (especially the tensor and spin-orbit forces) are included. Of course, the extended versions of the GBE CQM are now also better amenable to further studies, such as the N - N interaction (cf., *e.g.*, the works by Bartz and Stancu [36]). The different force components needed in these respects are directly brought about by the present GBE CQMs.

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